

Induced Gravity in Superfluid ^3He

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The gapless fermionic excitations in superfluid $^3\text{He-A}$ have the "relativistic" spectrum close to the gap nodes. This allowed us to model the modern cosmological scenarios of baryogenesis and magnetogenesis. The same massless fermions induce another low-energy property of the quantum vacuum – the gravitation. The effective metric of the space, in which the free quasiparticles move along geodesics, is not generally flat. Different order parameter textures correspond to curved effective space and produce many different exotic metrics, which are theoretically discussed in quantum gravity and cosmology. This includes the condensed matter analog of the black hole and event horizon, which can be realized in the moving soliton. This will allow us to simulate and thus experimentally investigate such quantum phenomena as the Hawking radiation from the horizon, the Bekenstein entropy of the black hole, and the structure of the quantum vacuum behind the horizon. One can also simulate the conical singularities produced by cosmic strings and monopoles; inflation; temperature dependence of the cosmological and Newton constants, etc.

PACS numbers: 67.57.-z, 04.60.-m, 04.70.-s, 11.27.+d, 98.80.Cq

1. Introduction

The quantum physical vacuum – the former ether – appears to be a complicated substance. Our present experimental physics is able to investigate only its long-wave-length properties. The highest energy of the elementary particles, i.e. of the elementary excitations of the physical vacuum, which is achieved today, is much smaller than the characteristic Planck value $E_P = \sqrt{\hbar c^5/G}$, where c is a speed of light and G the Newton gravitational

G.E. Volovik

constant. Their wave lengths are correspondingly much larger than the Planck scale $r_P = \hbar c/E_P \sim 10^{-33}\text{cm}$, which characterizes the “microscopic” structure of the vacuum – the “Planck solid”¹ or “Planck condensed matter”. Thus we are essentially limited in probing the microscopics of vacuum.

On the other hand we know that the low-energy properties of different condensed matter substances (magnets, superfluids, crystals, superconductors, etc.) are robust, i.e. do not depend much on the details of the microscopic structure of these substances. The main role is played by symmetry and topology of condensed matter: they determine the soft (low-energy) hydrodynamic variables, the effective Lagrangian describing the low-energy dynamics, and topological defects. The microscopic details provide us only with the “fundamental constants”, which enter the effective Lagrangian, such as speed of sound, superfluid density, modulus of elasticity, magnetic susceptibility, etc.

According to this analogy, such properties of our world, as gravitation, gauge fields, elementary chiral fermions, etc., arise as a low-energy soft modes of the “Planck condensed matter”. At high energy (of the Planck scale) these soft modes disappear: actually they merge with the continuum of the high-energy degrees of freedom of the “Planck condensed matter” and thus cannot be separated anymore.

The main advantage of the condensed matter analogy is that in principle we know the condensed matter structure at any relevant scale, including the interatomic distance, which corresponds to the Planck scale. Thus the condensed matter can serve as a guide on the way from our present low-energy corner to the Planckian and trans-Planckian physics. In this sense the superfluid phases of ^3He , especially $^3\text{He-A}$, are of most importance: the low-energy degrees of freedom in $^3\text{He-A}$ do really consist of chiral fermions, gauge fields and gravity. That is why, though there is no one-to-one correspondence between the $^3\text{He-A}$ and the “Planck condensed matter”, many phenomena of quantum vacuum can be simulated in $^3\text{He-A}$.

In a previous QFS-97 talk² we discussed the possibility of using the superfluid phases of ^3He as a model for the investigation of a property of the quantum vacuum known as chiral anomaly, which leads to the nonconservation of the baryonic charge and thus to the possibility of generation of the baryonic asymmetry of our present Universe. The ^3He analogue of the chiral anomaly was verified in experiments with quantized vortices.³ After QFS-97 it became clear that yet another effect observed in $^3\text{He-A}$ – collapse of the normal-to-superfluid counterflow with formation of textures⁴ – is related to the phenomenon of the chiral anomaly.^{5,6} Moreover it appeared that this collapse completely reproduces one of the scenarios of the generation of galactic magnetic fields in the early Universe.^{7,8} Another direction

Induced Gravity in Superfluid ^3He

which was developed in the period between QFS-97 and QFS-98 is related to the gravitation: how the gravity can be viewed from the ^3He -A physics and how different problems of gravity can be experimentally simulated in ^3He -A.

2. Gap Nodes and Gravity

The effective gravity, as a low-frequency phenomenon, arises in many condensed matter systems. In crystals, the effective curved space, in which “elementary particles” – phonons and Bloch electrons – are propagating, is induced by the distributed topological defects, dislocations and disclinations, (see e.g.^{9,10,11}). In the normal (or superfluid) liquids, the effective Lorentzian space for propagating sound waves (phonons) is generated by the hydrodynamic (super) flow of the liquid.^{12,13} However it appears that the superfluid ^3He -A provides us with the most adequate analogy of the relativistic theory of the effective gravity, first introduced by Sakharov.¹⁵

The most important property of the superfluid ^3He -A, as compared to other quantum fluids and solids, is that its elementary fermions – Bogoliubov-Nambu quasiparticles – are gapless. Their energy spectrum $E(\mathbf{p})$ contains the point nodes, where the energy is zero. Close to the gap nodes, i.e. at low energy, these quasiparticles have a well defined chirality: the quasiparticles are either left-handed or right-handed. This effectively reproduces our low-energy world (however above the electroweak energy scale), where all the fermionic elementary particles (electrons, neutrinos, quarks) are chiral. The point gap nodes are topologically stable and thus do not disappear under the deformation of the system, the gap nodes with opposite topological charges can however annihilate each other.⁶ If the total topological charge of the nodes is nonzero some of the gapless (massless) fermions will survive under any perturbation of the system. This algebraic conservation of massless fermions induced by the momentum space topology can be a good reason for the zero mass of neutrino: If we live in the vacuum with nonzero total topological charge, we should not spend time and money to look for the neutrino mass. Recent Kamiokande experiments have shown evidence of neutrino oscillations and thus the possibility of the neutrino mass.¹⁶ If this is true, this would mean that the total topological charge of the nodes in the “Planck condensed matter” is zero. We shall see.

Another interesting consequence of the existence of the gap node is that it immediately introduces the effective gravitational field. Let the node be situated at point $\mathbf{p}^{(0)}$ in momentum space, i.e. $E(\mathbf{p}^{(0)}) = 0$. If the space parity is not violated, then in the vicinity of the node the function $E^2(\mathbf{p})$

G.E. Volovik

takes a quadratic form for deviations from $\mathbf{p}^{(0)}$:

$$E^2(\mathbf{p}) = g^{ik}(p_i - p_i^{(0)})(p_k - p_k^{(0)}) . \quad (1)$$

The quantity g^{ik} plays the part of the effective metric tensor of the gravitational field, while $\mathbf{A} = \mathbf{p}^{(0)}$ corresponds to the vector potential of effective electromagnetic field. The other dynamical components of these fields appear, if the space parity is violated, for example if the superflow velocity $\mathbf{v}_s \neq 0$. In this case the quasiparticle energy is Doppler shifted, $E(\mathbf{p}) \rightarrow E(\mathbf{p}) + \mathbf{p} \cdot \mathbf{v}_s = E(\mathbf{p}) + \mathbf{p}^{(0)} \cdot \mathbf{v}_s + (\mathbf{p} - \mathbf{p}^{(0)}) \cdot \mathbf{v}_s$. This leads to appearance of the scalar vector potential $A_0 = \mathbf{p}^{(0)} \cdot \mathbf{v}_s$ and the mixed component of the metric tensor $g^{i0} = v_s^i$. With these identifications, Eq.(1) becomes fully “relativistic”:

$$g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = 0 . \quad (2)$$

Here the “electric charge”, $e = \pm$, reflects the fact that in $^3\text{He-A}$ each gap node has a sparring partner with an opposite momentum, $-\mathbf{p}^{(0)}$: quasiparticles near the opposite nodes have opposite charges (and also the opposite chiralities).

From the topological stability of the gap node in fermionic spectrum, it follows that the small deformations of the quantum vacuum do not destroy the gap node, but they lead to the variation of the fields $g^{\mu\nu}$ and A_μ . This means that the “gravitational metric” and the “gauge fields” are dynamical collective modes of the quantum vacuum in fermionic condensed matter which has the point gap nodes. Moreover, in the vicinity of the gap nodes the fermions are chiral and are described by the Weyl equations. Thus in the low-energy corner the $^3\text{He-A}$ has all the ingredients of the quantum field theory of our relativistic vacuum: chiral fermions, gauge fields and gravitation. This suggests that the “Planck condensed matter” belongs to the same universality class as $^3\text{He-A}$ (though not all the components of the gauge and graviational fields are independent in $^3\text{He-A}$, see⁶). Within this picture, the internal symmetries, such as $SU(2)$ and $SU(3)$, are consequences of the given number of gap nodes and the symmetry relations between them: In $^3\text{He-A}$ the $SU(2)$ gauge field naturally arises in this manner (see¹⁷). The main difference from the ideologically similar relativistic theories of induced gravity (such as in Ref.¹⁸ where the low-energy gravity is induced by massive relativistic quantum fields) is that in our case the gravity appears in the low-energy corner simultaneously with “relativistic” fermions and bosons.

3. Charge of Vacuum and Cosmological Term

An equilibrium homogeneous ground state of condensed matter has zero charge density, if charges interact via long range forces. For example, elec-

Induced Gravity in Superfluid ^3He

neutrality is the necessary property of bulk metals and superconductors; otherwise the vacuum energy of the system diverges faster than its volume. Another example is the algebraic density of quantized vortices in superfluids, $\langle \nabla \times \mathbf{v}_s \rangle$: due to logarithmic interaction of vortices it must be zero in equilibrium homogeneous superfluids (in the absence of external rotation). The same argument can be applied to the “Planck condensed matter”, and it seems to work. The density of the electric charge of the Dirac sea is zero due to exact cancellation of electric charges of electrons, q_e , and quarks, q_u and q_d , in the fermionic vacuum: $Q_{\text{vac}} = \sum_{E<0} (q_e + 3q_u + 3q_d) = 0$. In Einstein theory the energy-momentum tensor of the vacuum should be a source of the long range gravitational forces. The immediate consequence is that for the equilibrium homogeneous state of the vacuum one has $\partial S_{\text{vac}}/\partial g_{\mu\nu} = \sqrt{-g}T_{\text{vac}}^{\mu\nu} = 0$ (this equilibrium condition contains partial derivative, instead of functional derivative). This leads to nullification of the cosmological term in Einstein equations for vacuum in equilibrium, which agrees with the experiment. But it is in very serious disagreement with theoretical estimations of fermionic or bosonic zero-point vacuum energy, which is 120 orders of magnitude higher than the experimental upper limit. What is wrong with theory?

The situation is probably similar to what we have in condensed matter if we want to estimate the ground-state energy from the zero-point energy of phonons: $E_{\text{zp}} = (1/2) \sum_{\mathbf{k}} \hbar\omega(\mathbf{k})$. This never gives the correct estimation of the vacuum energy; moreover sometimes it has even the wrong sign. This occurs because the phonon modes are soft variables and are determined only in the low-energy limit, whereas the vacuum energy of the solid is determined by the quantum many-body physics, where essentially the high-energy degrees of freedom are involved. These high-energy degrees are always adjusted to provide the electroneutrality, irrespective of the low-energy physics. This suggests two consequences:

(1) The ^3He analogy suggests that a zero value of the cosmological term in the equilibrium vacuum is dictated by the Planckian or trans-Planckian degrees of freedom: $\partial S_{\text{vac}}/\partial g_{\mu\nu} = 0$ is the thermodynamic equilibrium condition for the “Planck condensed matter”. Thus the equilibrium homogeneous vacuum does not gravitate. Deviations of the vacuum from its equilibrium can gravitate.¹⁹ For example, in the presence of matter, a small cosmological term of the order of the energy density of the matter is possible.

(2) The quantization of the low-energy degrees of freedom, such as the gravitational field, works only in the low-energy limit, but cannot be applied to higher energies. Moreover, even in the low-energy limit the quantization leads to the double counting: The total energy of the solid (and the same can be applied to the “Planck solid”) is obtained by the solution of the

G.E. Volovik

quantum many-body problem, which automatically takes into account all the degrees of freedom. If one now separates the soft modes and adds the zero-point energy of these modes, this will be the double counting. The gravity is a low-frequency, and actually classical result of quantization of high-energy degrees of freedom of the “Planck condensed matter”, so one should not quantize the gravity again. The quantization of gravity is similar to the attempt to derive the microscopic structure of the solid, using only the spectrum of the low-energy phonons. The $^3\text{He-A}$ analogy also suggests that all the degrees of freedom, bosonic and fermionic, can come from the initial (bare) fermionic degrees of freedom.

4. Gravitational Constant in $^3\text{He-A}$.

$^3\text{He-A}$ is not a perfect system for the modelling of all the aspects of gravity. The effective Lagrangian for $g_{\mu\nu}$, which is obtained after integration over the fermionic degrees of freedom, contains many noncovariant terms (see discussion in^{6,20}). However some terms correspond to the Einstein action, and this allows us to estimate the gravitational constant G in the effective theory. Consider an analog of the graviton field in $^3\text{He-A}$. In the relativistic gravitation the energy density of the graviton propagating along z direction is written in terms of the perturbation of the metric, $g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$:

$$\mathcal{T}_0^0 = \frac{1}{16\pi G} \left[(\partial_z h_{xy})^2 + \frac{1}{4} ((\partial_z (h_{xx} - h_{yy}))^2 \right] . \quad (3)$$

In $^3\text{He-A}$ such a perturbation corresponds to the so-called clapping mode with spin 2.^{17,20} The clapping mode and the graviton have the same structure of the energy density; thus comparing coefficients in expressions for \mathcal{T}_0^0 one obtains the temperature dependent effective gravitational constant:²⁰

$$G(T) = \frac{12\pi}{K(T)\Delta^2(T)} . \quad (4)$$

Here Δ is the gap amplitude, which plays the part of the Planck energy; $K(T)$ is a dimensionless function of T , which close to the superfluid transition temperature T_c is

$$K(T) = 1 - \frac{T^2}{T_c^2} , \quad T \rightarrow T_c \sim \Delta(0) . \quad (5)$$

The temperature dependence of G corresponds to screening of gravity by the fermionic vacuum. The dependence coming from the factor $K(T)$ is the traditional one: Since the effective action for gravity is obtained from the integration over the fermionic (or bosonic) degrees of freedom,¹⁵ it is

Induced Gravity in Superfluid ^3He

influenced by the thermal distribution of fermions. What is new here is that G is also influenced by the temperature dependence of the “Planck” energy cut-off $\Delta(T)$, which is determined by details of the trans-Planckian physics. In $^3\text{He-A}$, $\Delta^2(T) \sim \Delta^2(0)(1 - T^2/T_c^2)$. This provides an illustration of how the corrections T^2/E_P^2 , caused by the Planckian physics, come into play even at low T .¹ So, we can hope, that even the present “low-energy” physics contains the measurable corrections coming from the Planck physics.

5. Event Horizon

At the classical level the existence of an event horizon leads to the divergency of the density of the particle states at the horizon (see e.g.²¹ and references therein). At the quantum level this is believed to give rise to the Bekenstein entropy of the black hole, and the Hawking radiation from the black hole.^{22,23} There are many problems related to these issues: Are the entropy and the Hawking radiation physical? Does the entropy come from degrees of freedom (i) outside the horizon, (ii) on the horizon, (iii) inside the horizon? Since the particle momentum grows up to the Planck scale at the horizon, trans-Planckian physics is inevitably involved. Also it is not completely clear whether the quantum vacuum is stable in the region behind horizon.

These problems can be treated in condensed matter of the $^3\text{He-A}$ type, where in the “cis-Planckian” scale the quasiparticles – excitations of the quantum vacuum – are “relativistic” and obey the dynamical equations, determined by the effective Lorentzian metric in Eq.(2). Existence of the quantum vacuum, which can respond to the change of the fermionic spectrum in the presence of horizon, is instrumental and represents a big advantage of $^3\text{He-A}$ compared with conventional liquids and other dissipative systems,^{12,13,14} where the event horizon can also arise.

5.1. Horizon within moving domain wall.

In $^3\text{He-A}$ the analog of event horizon appears in the moving topological soliton.²⁴ Here we consider soliton moving in a thin film of $^3\text{He-A}$. In the parallel-plane geometry the unit vector $\hat{\mathbf{l}}$, which determines the direction of the gap node in momentum space, is fixed by the boundary conditions – it should be normal to the film: $\hat{\mathbf{l}} = \pm \hat{\mathbf{z}}$ if the film is in the plane x, y plane. Due to the double degeneracy, a topologically stable domain wall can exist which separates two half-spaces with opposite directions of $\hat{\mathbf{l}}$. The classical spectrum of the low-energy fermionic quasiparticles in the presence of the domain wall is:

$$E^2(\mathbf{p}) = (c^z)^2(p_z \mp p_F)^2 + (c^x)^2 p_x^2 + (c^y)^2 p_y^2 . \quad (6)$$

G.E. Volovik

In the simplest soliton the “speeds of light” propagating along x, y, z are

$$c^z = v_F, \quad c^y = c_\perp, \quad c^x(x) = c_\perp \tanh \frac{x}{d}, \quad c_\perp = \frac{\Delta(T)}{p_F}, \quad (7)$$

where p_F and v_F are the Fermi momentum and Fermi velocity; the thickness d of the domain wall is on order of superfluid coherence length, $d \sim \xi \sim v_F/\Delta(T)$. In this domain wall the “speed of light” propagating across the wall, $c^x(x)$, changes sign, while the other two remain constant.

If such a domain wall is moving with velocity v along x , the Doppler shifted fermionic spectrum gives the following time-independent metric in the frame of the moving wall:

$$g^{zz} = v_F^2, \quad g^{yy} = c_\perp^2, \quad g^{xx} = (c^x(x))^2 - v^2, \quad g^{00} = -1, \quad g^{0x} = v. \quad (8)$$

The interval corresponding to this effective metric is obtained from the inverse metric $g_{\mu\nu}$:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 \left(1 - \frac{v^2}{(c^x)^2} \right) - 2dxdt \frac{v}{(c^x)^2} + \frac{dx^2}{(c^x)^2} + \frac{dy^2}{c_\perp^2} + \frac{dz^2}{v_F^2}. \quad (9)$$

Two event horizons occur at points $x = \pm x_h$, where both $g^{xx} = 0$ and $g_{00} = 0$,

$$\tanh \frac{x_h}{d} = \frac{v}{c_\perp}. \quad (10)$$

At these points the speed of particles in the x -direction equals the velocity of the soliton in the same direction, $|c^x(\pm x_h)| = v$, Fig. 1. Between the horizons, $|c^x|$ is less than v , thus particles cannot propagate out across the horizon at $x = x_h$. This horizon represents the black hole. Another horizon, at $x = -x_h$, corresponds to the white hole, since the particles cannot propagate into the region behind horizon.

So, this domain wall represents the nonrotating black hole with the singularity at $x = 0$. Moreover, as distinct from the extended description of the black hole, both past and future horizons are physical. In condensed matter the event horizon appears only if the effective metric is non-static, i.e. $g_{0i} \neq 0$. Is this constraint valid for the Planck condensed matter too?

5.2. Hawking temperature and Bekenstein entropy

In the presence of a horizon the vacuum becomes ill-defined. This can be seen from thermodynamic functions of “relativistic” fermions in the moving soliton. For example, let us consider the normal component of the liquid (“matter”), which comoves with the soliton and has temperature T . Then

Induced Gravity in Superfluid ^3He

the local pressure of this system²⁵

$$P = \hbar \frac{7\pi^2}{180} \frac{T^4}{g_{00}^2 \hbar^4} (-g)^{1/2} = \frac{7\pi^2}{180 \hbar^3} \frac{\tilde{T}^4}{v_F c_\perp c^x} \quad , \quad \tilde{T} = T \left(1 - \frac{v^2}{(c^x)^2} \right)^{-1/2} \quad , \quad (11)$$

diverges at horizon together with the effective temperature \tilde{T} . Thus there is no equilibrium thermodynamic state in the presence of horizon. The existence of a horizon is associated with the dissipative state characterized by the Hawking radiation. The latter is thermal black-body radiation, with temperature determined by the “surface gravity”:

$$T_H = \frac{\hbar}{2\pi k_B} \kappa \quad , \quad \kappa = \left(\frac{dc^x}{dx} \right)_h \quad . \quad (12)$$

In our case the Hawking temperature depends on the velocity v :

$$T_H(v) = T_H(v=0) \left(1 - \frac{v^2}{c_\perp^2} \right) \quad , \quad T_H(v=0) = \frac{\hbar c_\perp}{2\pi k_B d} \quad (13)$$

The Hawking radiation can be detected by quasiparticle detectors. Also it leads to the energy dissipation and thus to deceleration of the moving domain wall even if the real temperature $T = 0$. It would be interesting to find out whether the dissipation disappears and the velocity of the domain wall is stabilized if the real temperature of superfluid equals the Hawking temperature, $T = T_H(v)$. Some hint on such a possibility can be found in the paper on the moving boundary between $^3\text{He-A}$ and $^3\text{He-B}$.²⁶

Due to deceleration caused by Hawking radiation, the Hawking temperature increases with time. The distance between horizons, $2x_h$, decreases until the complete stop of the domain wall when the two horizons merge. In a similar manner, shrinking of the black hole can stop after the Planck scale is reached. The Hawking temperature approaches its asymptotic value $T_H(v=0)$ in Eq.(13), but when the horizons merge, the Hawking radiation disappears, since the stationary domain wall is nondissipative. One can show,²⁷ that the entropy of the domain wall does not disappear, but approaches a finite value, which corresponds to one degree of quasiparticle freedom per Planck area. This is similar to the Bekenstein entropy, but it comes from the “nonrelativistic” physics at the “trans-Planckian” scale. Nonzero entropy results from the fermion zero modes: bound states at the domain wall with exactly zero energy. Such bound states, dictated by topology of the texture, are now intensively studied in high-temperature superconductors and other unconventional superconductors/superfluids (see references in²⁸ and²⁹). Event horizons with Hawking radiation and Bekenstein entropy also appear in the core of vortices.³⁰ The influence of the high-energy non-relativistic spectrum on Hawking radiation is under investigation (see³¹).

G.E. Volovik

6. Metric Induced by Quantized Vortices

The linear and point-like topological defects also induce effective metrics, which can be interesting for the theory of gravitation. The simulation of 2D and 3D conical singularities by disgyrations and monopoles in $^3\text{He-A}$ one can find in Ref.²⁰ Here we consider quantized vortices in superfluid $^3\text{He-A}$ films.

6.1. Vortex as cosmic spinning string

The vortex in thin films, where the $\hat{\mathbf{l}}$ -vector is fixed, is a topologically stable object with the superfluid velocity circulating around the origin, $\mathbf{v}_s = N\kappa\hat{\phi}/2\pi r$. Here $\kappa = \pi\hbar/2m_3$ is the quantum of circulation (m_3 is the mass of the ^3He atom), and N is an integer or half-odd integer circulation quantum number. This superflow induces the effective space, where fermions are propagating along geodesic curves, with the interval³²

$$ds^2 = - \left(1 - \frac{v_s^2(r)}{c_\perp^2} \right) dt^2 - \frac{\hbar N}{m_3 c_\perp^2} d\phi dt + \frac{1}{c_\perp^2} (dr^2 + r^2 d\phi^2) + \frac{1}{v_F^2} dz^2. \quad (14)$$

Far from the vortex axis, where $v_s \ll c_\perp$, this transforms to the metric induced by the cosmic spinning string. This similarity leads to the gravitational Aharonov-Bohm effect and as a result to an Iordanskii type lifting force acting on the vortex (in the presence of quasiparticles) and on the spinning string (in the presence of matter).³²

6.2. Instability of ergoregion

Another important property of the metric induced by the vortex is the existence of the so-called ergoregion, where quasiparticles have negative energy: $E(\mathbf{p}) + \mathbf{p} \cdot \mathbf{v}_s < 0$. The ergoregion occupies a cylinder of radius $r_e = \hbar N / 2m_3 c_\perp$, where the flow velocity is larger than the speed of light, $v_s > c_\perp$, and thus $g_{00} < 0$ (Fig.2a). The flow is circulating along the boundary of the ergoregion, so the horizon is absent here: the ergosurface is thus separated from the horizon. A similar situation occurs for the rotating black hole; and it was argued that the ergoregion can be unstable there³³ Here we provide a condensed matter illustration of such an instability. It was shown³⁴ that for the superfluids with “relativistic” fermions, such as superfluid $^3\text{He-A}$, the superfluid vacuum is unstable if v_s exceeds the “speed of light”. In the vortices with small N such an instability is prevented by the gradients of the order parameter in the core. But if the vortex winding number is big, $N \gg 1$, so that the radius of the ergoregion essentially exceeds the superfluid coherence length, $r_e = \hbar N / 2m_3 c_\perp \gg \hbar / m_3 c_\perp \sim \xi$, the instability can develop.

Induced Gravity in Superfluid ^3He

The development of the vacuum instability is also an interesting phenomenon. In cosmology it can result in the inflationary expansion of the Universe. In principle we can also construct the situation in which the “speed of light” decays exponentially or as a power law faster than $1/t$, and thus simulate the inflation and investigate the growth of perturbations during inflation.

In the case of the vortex with $N \gg 1$, the vacuum instability in the ergoregion results in the splitting of the N -quantum vortex into N singly quantized vortices. However it is interesting to consider what happens if we impose the condition that the axial symmetry of the vortex is preserved. In this case the resulting equilibrium core structure is shown in Fig.2b. The thin shell, with the thickness of order the coherence length (Planck length), which consists of the normal (non-superfluid) state, separates two superfluid vacua. In the inner vacuum the superflow is absent; as a result the metric experiences a jump across the shell. The shell is situated at the radius $r_c > r_e$, so that the ergoregion is completely erased: both inner and outer vacua do not contain ergoregions. Thus an equilibrium vacuum does not sustain ergoregions.

Another problem is related to dependence of the vacuum on the reference frame. Due to Galilean invariance, there are two sources of the same non-static metric, say, in Eq.(14). One of them is the superfluid velocity field $\mathbf{v}_s(\mathbf{r})$; another one appears in the moving texture in the comoving frame. The vacuum is the same for two metrics, unless a horizon or an ergoregion appear, similar to ambiguity of relativistic quantum vacuum in the presence of horizon. This problem includes properties of rotating quantum vacuum (see e.g.³⁵) and can be investigated in $^3\text{He-A}$ using rotating cylinders.

7. Conclusion

Using superfluid phases of ^3He one can simulate a broad spectrum of phenomena related to the quantum vacuum. We discussed only few of them. On the conceptual level the lessons of $^3\text{He-A}$ suggest: The gravity is the property of quantum “Planck matter” in the classical low-energy limit. Gravitational field arise as collective modes of the dynamical deformations of topologically stable gap nodes. The same topological stability provides the zero mass of neutrino, if the overall topological invariant of the “Planck matter” is nonzero. The equilibrium state of “Planck matter” does not gravitate, suggesting possible route to solution of cosmological constant problem. The vacuum can be highly unstable behind horizon, and thus can resist to formation of black holes. Fundamental constants G and c should depend on temperature; what about the third fundamental constant – Planck constant

G.E. Volovik

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Induced Gravity in Superfluid ^3He

Figure Captions:

1. Solid line is the “speed of light” in the x -direction, $c^x(x)$, for the domain wall moving with velocity v in thin film of ^3He -A. The speed of light crosses zero at $x = 0$. For the moving wall the black and white hole pair appears for any velocity v below $|c^x(\infty)|$. At horizons $g^{xx}(\pm x_h) = 0$ and $g_{00}(\pm x_h) = 0$. Arrows show possible directions of propagation of particles.

2. (a) Regular structure of the core of the vortex. In the region $r < r_e$ the superfluid velocity v_s exceeds the speed of “light”, c_\perp , and $g_{00} > 0$. In this ergoregion the quasiparticle energy is negative. For vortices with large winding number $N \gg 1$, the quantum superfluid vacuum is unstable in the ergoregion. (b) For the axially symmetric vortex this instability leads to the reconstruction of the core. In the new core structure, the normal shell separates two superfluid vacua, in which superfluid velocity does not exceed the speed of “light”. Ergoregion is absent.

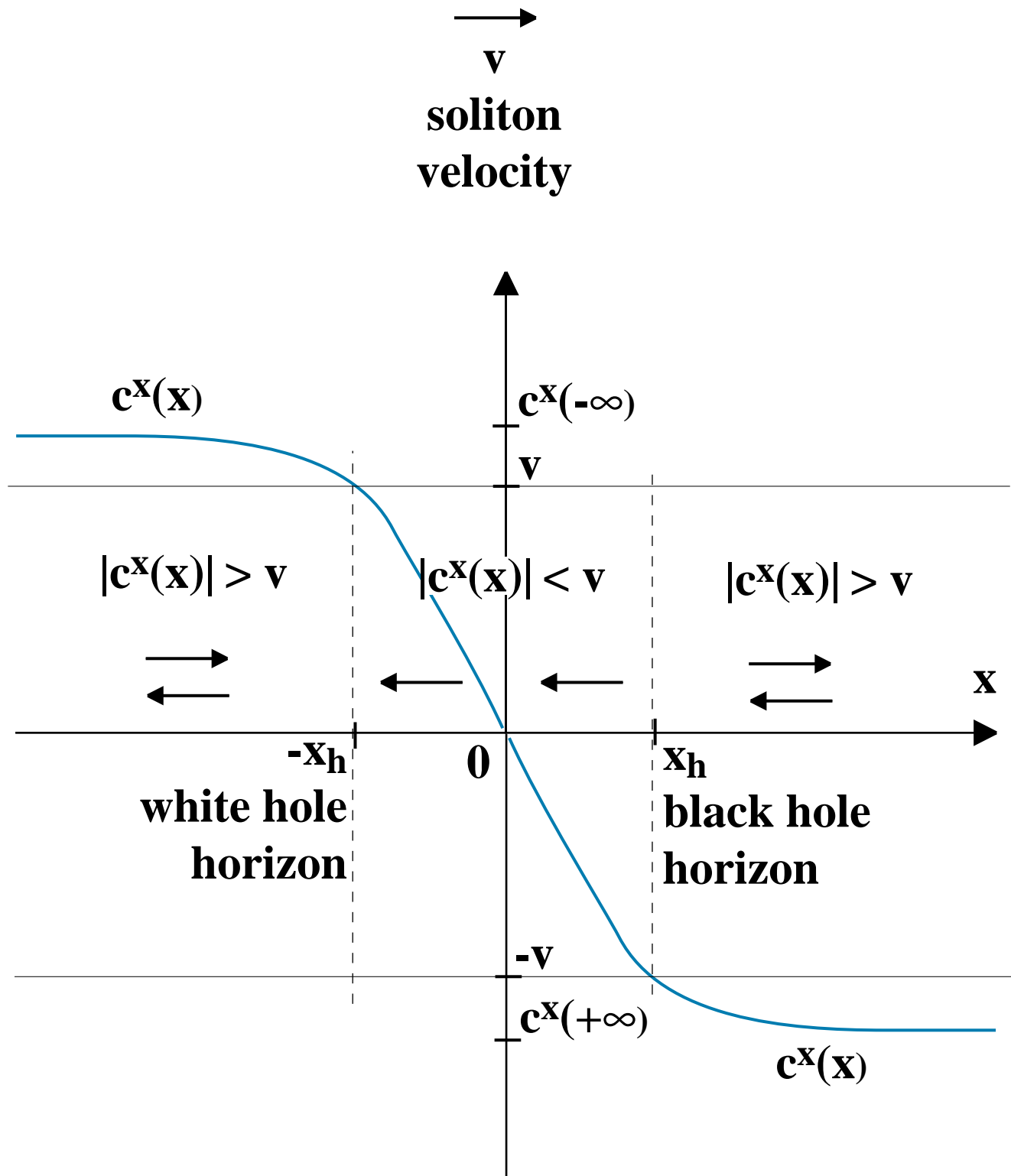
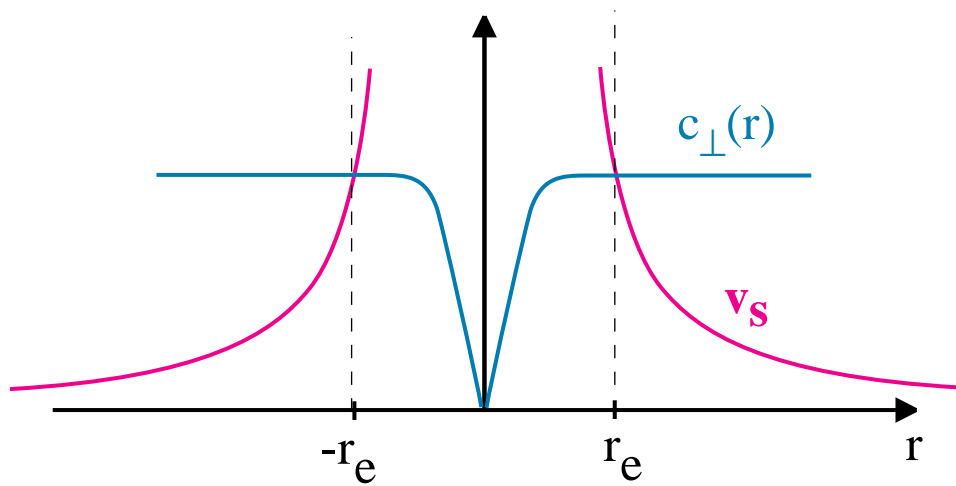
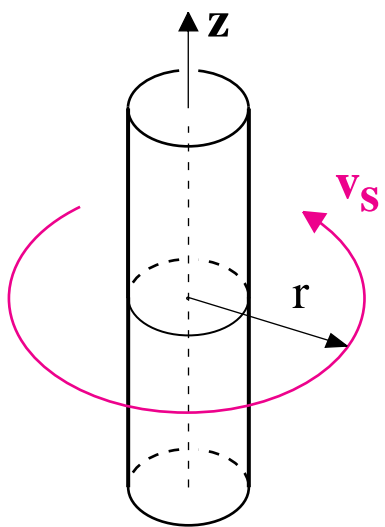
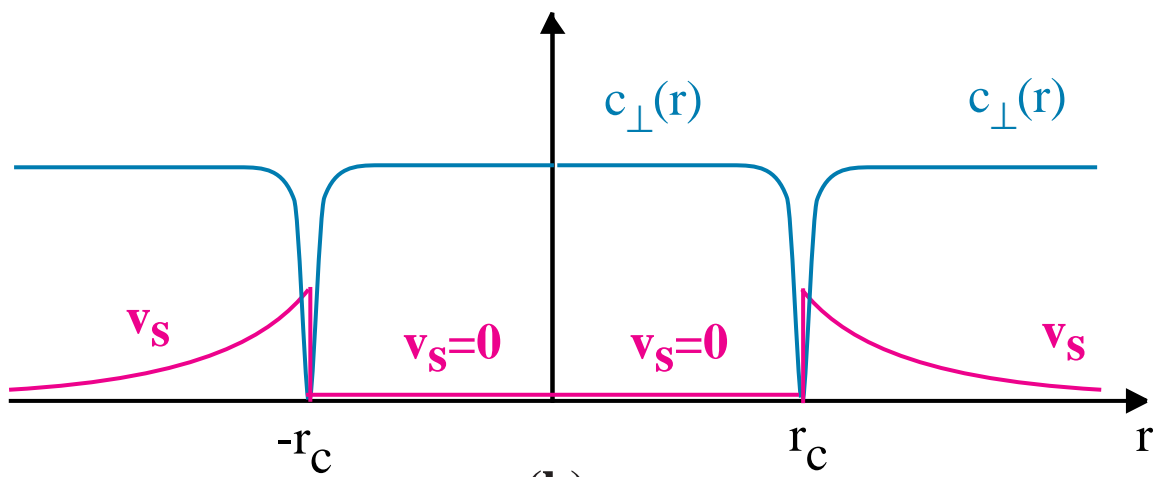


Fig.1



(a)



(b)

Fig.2